



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/32

Paper 3 Further Mechanics

October/November 2025

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

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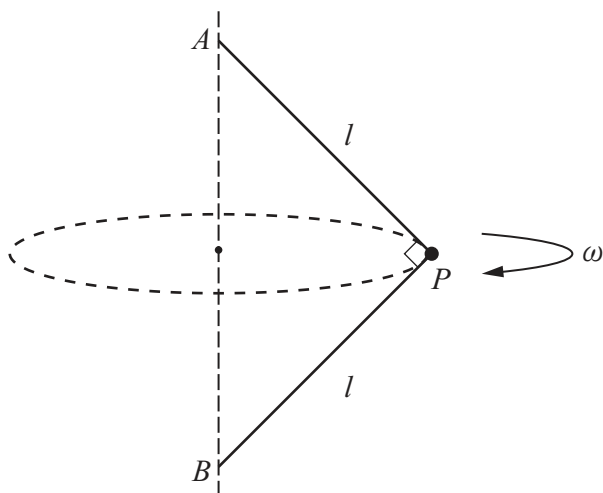
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- 1 A particle P of mass m is attached to two light inextensible strings each of length l . The end of one string is attached to a fixed point A and the end of the other string is attached to a fixed point B , with A vertically above B . Angle APB is a right angle. The particle P rotates in a horizontal circle at a constant angular speed ω with both strings taut (see diagram).



Find the tension in string AP in terms of m , g , l and ω .

[4]

[illegible]



- 2 One end of a light elastic string of natural length a and modulus of elasticity $2mg$ is attached to a fixed point A on a rough horizontal surface. The other end of the string is attached to a particle P of mass m . The particle and string rest on the surface. The coefficient of friction between P and the surface is μ . The particle P is initially held in equilibrium at a distance $\frac{4}{3}a$ from A . The particle is then released from rest.

(a) Given that the string never becomes slack, find the minimum value of μ . [3]

[illegible]



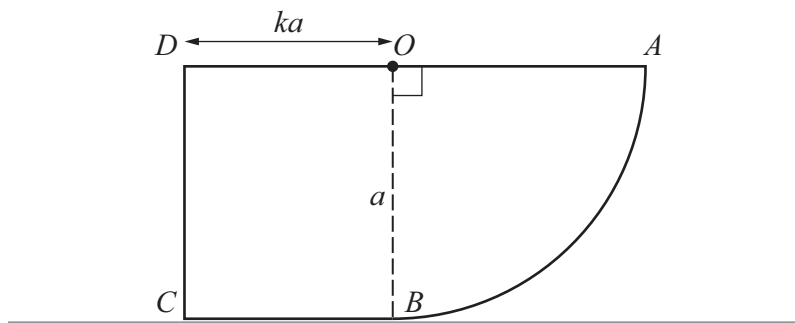
It is now given that $\mu = \frac{1}{2}$.

(b) Find the extension of the string when the particle comes to rest.

[3]

[illegible]

- 3 A uniform lamina $OABCD$ is in the form of a rectangle, $OBCD$, joined along the edge OB to a quarter circle OAB . The length of DO is ka and the length of OB is a . The lamina rests in a vertical plane with its edge CB on a horizontal surface (see diagram).



- (a) Find, in terms of k , a and π , an expression for the distance of the centre of mass above the horizontal surface. [4]

[You may use without proof the result for the centre of mass of a circular sector in the list of formulae (MF19).]

[illegible]



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The lamina is on the point of toppling about B .

- (b) Find the value of k . [2]

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- 4 A particle Q is initially positioned at a distance d vertically above a particle P . Particle P is projected with speed U at an angle α above the horizontal. At the same time, Q is projected at an angle β below the horizontal. Both particles move freely under gravity. The particles collide at time T after the projections.

(a) Show that $d = UT(\sin \alpha + \cos \alpha \tan \beta)$. [4]

[illegible]



The particles collide when P is at its maximum height.

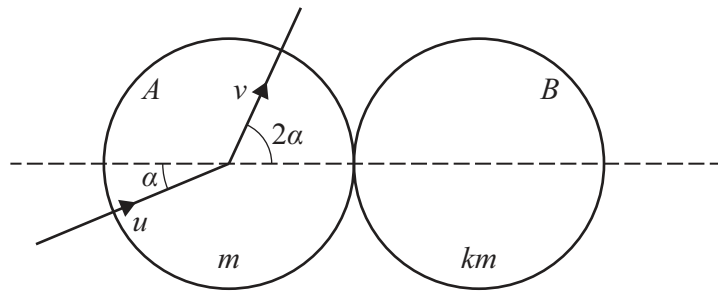
- (b)** Given that $\alpha = 30^\circ$ and $\beta = 60^\circ$, find d in terms of U and g .

[3]

[illegible]



- 5 Two uniform smooth spheres, A and B , of equal radii are on a horizontal surface. They have masses of m and km respectively. Sphere A is moving with speed u at an angle α with the line of centres when it collides with sphere B which is stationary. Immediately after the collision, sphere A moves with speed v at an angle 2α with the line of centres (see diagram).



It is given that $\tan \alpha = \frac{3}{4}$.

- (a) Find v in terms of u .

[2]

[illegible]

- (b)** Find the coefficient of restitution between the spheres in terms of k .

[4]

[illegible]



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(c) Find the range of possible values of k . [3]

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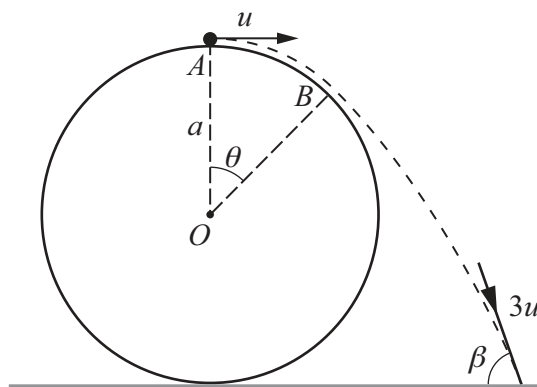
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A fixed smooth sphere with radius a and centre O rests on horizontal ground. A particle is projected horizontally from the highest point, A , of the sphere with speed u . The particle begins to move in a vertical circle along the surface of the sphere. The particle loses contact with the sphere at the point B , where the angle AOB is θ .

After leaving the surface of the sphere, the particle moves freely under gravity before striking the horizontal ground with speed $3u$ at an angle β to the horizontal (see diagram).

Find the value of β .

[8]

[illegible]

[illegible]



- 7 A particle P of mass m kg moving along a rough horizontal table has displacement x m from a fixed point O on the table and velocity v m s⁻¹ at time t s. The particle P is subject to a resistive force of magnitude $mgkv$ N, where k is a positive constant, and a frictional force of magnitude μmg . The particle P is initially at O with speed U m s⁻¹.

(a) Show that $t = \frac{1}{gk} \ln \left(\frac{kU + \mu}{kv + \mu} \right)$. [4]

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It is given that $U = 10$, $k = 0.04$ and $\mu = 0.2$.

- (b)** Find the distance P moves before coming to rest.

[4]

[illegible]

- (c) Find the average speed of P over the period it is moving.

[2]

[illegible]



Additional page

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